EE 362: Signals and Systems Name: Solutions

Spring 2019

Homework #7 (Due: Thursday, March 14, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

(1) A continuous-time signal $x(t)$ has Fourier transform

$$X(\omega) = \frac{1}{j\omega + b}$$

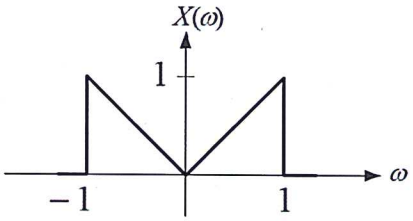
where b is a constant. Determine Fourier transform $V(\omega)$ of the following signals: [2 points each; 6 points total]

(a) $v(t) = x(-t)$

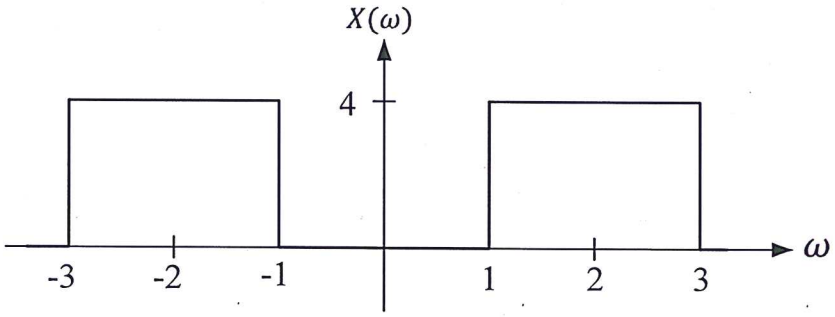
(b) $v(t) = \frac{dx(t)}{dt} + 3\frac{d^2x(t)}{dt^2}$

(c) $v(t) = x(t) * x(t)$

(2) Compute the inverse Fourier transform of the following frequency functions. [8 points each; 16 points total]



(a)



(b)

"Solutions"

(1)

a) $v(t) = x(-t)$

$$V(w) = X(w)^* = \frac{1}{b - jw}$$

b) $$v(t) = \frac{dx(t)}{dt} + 3\frac{d^2x(t)}{dt^2}$$

$$V(w) = (jw)\left(\frac{1}{b+jw}\right) + 3(jw)^2\left(\frac{1}{b+jw}\right)$$

$$= \frac{jw}{b+jw} - \frac{3w^2}{b+jw}$$

c) $v(t) = x(t) * x(t)$

$$V(w) = X(w)X(w) = \frac{1}{(b+jw)^2}$$

(2) -a

$$X(w) = P_2(w) - \left(1 - \frac{2|w|}{2}\right)P_2(w)$$

$$P_2(t) \leftrightarrow 2\,\text{sinc}\left(\frac{2w}{2\pi}\right)$$

$$\swarrow \quad \searrow$$

$$2\,\text{sinc}\left(\frac{t}{\pi}\right) \leftrightarrow 2\pi P_2(w)$$

OR $\frac{1}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right) \leftrightarrow P_2(w)$ — (a)

$$\left(1-\frac{2|t|}{2}\right)P_2(t) \leftrightarrow \frac{2}{2}\,\text{sinc}^2\left(\frac{2\omega}{4\pi}\right)$$

$$\text{sinc}^2\left(\frac{t}{2\pi}\right) \leftrightarrow 2\pi\left(1-\frac{2|\omega|}{2}\right)P_2(\omega)$$

$$\frac{1}{2\pi}\,\text{sinc}^2\left(\frac{t}{2\pi}\right) \leftrightarrow \left(1-\frac{2|\omega|}{2}\right)P_2(\omega) \quad -(b)$$

From (a) and (b):

$$x(t) = \frac{1}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right) - \frac{1}{2\pi}\,\text{sinc}^2\left(\frac{t}{2\pi}\right), \quad -\infty < t < \infty$$

(2)-b

$$X(\omega) = 4P_2(\omega+2) + 4P_2(\omega-2) = 4\left[P_2(\omega+2) + P_2(\omega-2)\right]$$

$$P_2(t) \leftrightarrow 2\,\text{sinc}\left(\frac{\omega}{\pi}\right)$$

$$2\,\text{sinc}\left(\frac{t}{\pi}\right) \leftrightarrow 2\pi P_2(\omega)$$

$$\frac{1}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right) \leftrightarrow P_2(\omega)$$

$$\therefore \frac{1}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right)\cos(2t) \leftrightarrow \frac{1}{2}\left[P_2(\omega+2) + P_2(\omega-2)\right]$$

$$\frac{8}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right)\cos(2t) \leftrightarrow 4\left[P_2(\omega+2) + P_2(\omega-2)\right]$$

$$\therefore x(t) = \frac{8}{\pi}\,\text{sinc}\left(\frac{t}{\pi}\right)\cos(2t)$$